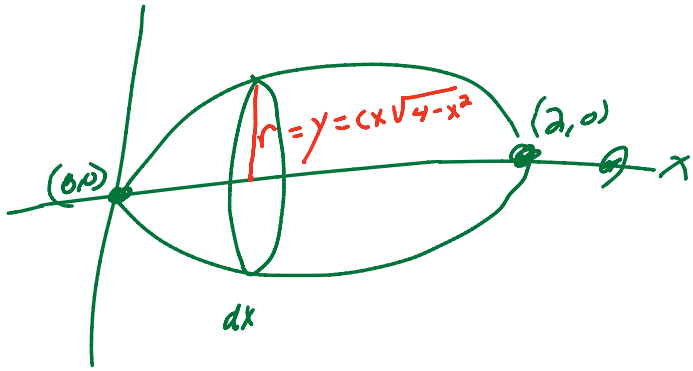
Period 1, May 10, 2024

$\pi r^2 = \text{Area}$

#3.

c.



$y = cx\sqrt{4-x^2}$

$0 = cx\sqrt{4-x^2}$

$c = +$

$x = \pm\sqrt{4} = \pm 2$

$cx\sqrt{4-x^2} \cdot cx\sqrt{4-x^2}$

$$2\pi = \int_0^2 \pi \cdot \left(cx\sqrt{4-x^2}\right)^2 dx$$

$$2\pi = \pi \int_0^2 \left(cx\sqrt{4-x^2}\right)^2 dx$$

$$2\pi = \pi \int_0^2 c^2x^2(4-x^2)\, dx$$

$$\frac{2\cancel{\pi}}{\cancel{\pi}} = \frac{\pi \int_0^2 \left[4c^2x^2 - c^2x^4\right] dx}{\cancel{\pi}}$$

$$2 = \frac{4c^2}{3} \cdot x^3 - \frac{c^2}{5}x^5 \Big[_0^2$$

$$2 = \frac{4c^2}{3}(2)^3 - \frac{c^2}{5}(2)^5 - \cancel{\left[\frac{4c^2}{3}(0)^3 - \frac{c^2}{5}(0)^5\right]}$$

$$2 = \frac{32c^2}{3} - \frac{32c^2}{5}$$

$$\frac{2}{\frac{32}{3} - \frac{32}{5}} = \frac{c^2\cancel{\left(\frac{32}{3} - \frac{32}{5}\right)}}{\cancel{\frac{32}{3} - \frac{32}{5}}}$$

$$\sqrt{\frac{2}{\frac{32}{3} - \frac{32}{5}}} = c$$

$$c = \sqrt{\frac{2}{\frac{32}{3} - \frac{32}{5}}}$$

a.

$$y = 6x\sqrt{4-x^2}$$

$$\int_0^2 6x\sqrt{4-x^2}\,dx = \int_4^0 6x\sqrt{u}\cdot\frac{du}{-2x} = -3\int_4^0 u^{\frac{1}{2}}\,du$$

$$u = 4-x^2 \Rightarrow du = -2x\,dx \Rightarrow \frac{du}{-2x} = dx$$

$$0 = 4-2^2 \qquad x=2$$

$$4 = 4-0^2 \qquad x=0$$

$$-\cancel{3}\cdot\frac{2}{\cancel{3}}\,u^{\frac{3}{2}}\Big[_4^0$$

$$\cancel{-2(0)^{\frac{3}{2}}} - \left[-2(4)^{\frac{3}{2}}\right]$$

$$+2\cdot 8 = 16$$

$$-2u^{\frac{3}{2}}\Big[_4^0$$

$$=$$

$$-2(4-x^2)^{3/2}\Big[_0^2$$

---

4. $G(x) = \int_0^x F(T)\,dT$

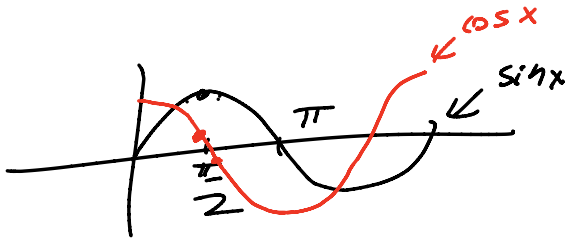
$G'(x) = F(x)$

$G''(x) = F'(x) = \text{Slope of } F(x)$

$G(x)$ is concave up when $F(x)$ has a + slope

$G(x)$ is concave Down when $F(x)$ has a − slope

5



$$\frac{dy}{dx} = \frac{y\cos x}{4y - \sin x}$$

$\cos\frac{\pi}{2} = 0$

x is a little less than $\frac{\pi}{2}$

$\cos x$ = positive

x is a little greater than $\frac{\pi}{2}$

$\cos x$ = negative

$(\frac{\pi}{2}, 0)$

$\sin\frac{\pi}{2} = 1$

x is a little less than $\frac{\pi}{2} \approx 1$

x is a little bigger than $\frac{\pi}{2} \approx 1$

$x < \frac{\pi}{2}$ $\quad \frac{dy}{dx} = \frac{2 \cdot +}{7} = +$

$(\frac{\pi}{2}, 2)$ $\quad \frac{dy}{dx} = \frac{y\cos x}{4y - \sin x}$

$8 - 1 = 7$

$x > \frac{\pi}{2}$ $\quad \frac{dy}{dx} = \frac{2 \cdot -}{7} = -$

---

6. $\ln|12-y| = -\frac{1}{3}T - C$

$\ln_e a = b$

$e^b = a$

$e^{-\frac{1}{3}T + C} = 12 - y$

$e^{-\frac{1}{3}T} \cdot e^{-C} = 12 - y$

$e^{-\frac{1}{3}T} \cdot k = 12 - y$

$-12 \quad -12$

$e^{-\frac{1}{3}T} \cdot k - 12 = -y$

$12 - e^{-\frac{1}{3}T} \cdot k = y$